WIND-TUNNEL INVESTIGATION OF EFFECT OF YAW
ON LATERAL-STABILITY CHARACTERISTICS
I - FOUR N.A.C.A. 23012 WINGS OF VARIOUS PLAN FORMS
WITH AND WITHOUT DIHEDRAL
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Four N.A.C.A. 23012 wings were tested at several angles of yaw in the N.A.C.A. 7- by 10-foot wind tunnel. All the wings have rounded tips and, in plan form, one is rectangular and the others are tapered 3:1 with various amounts of sweep. Each wing was tested with two amounts of dihedral and with partial-span split flaps.

The coefficients of lift, drag, and pitching moment are given for all the models at zero yaw. The coefficients of rolling moment, yawing moment, and side force are given for the rectangular wing at all values of yaw tested. The rate of change in the coefficients with angle of yaw is given in convenient form for stability calculations.

INTRODUCTION

Mathematical equations have been available for some time and convenient charts are given in reference 1 for predicting the lateral stability of airplanes. The practical use of the equations or charts, however, depends upon the knowledge of the lateral-stability derivatives for the particular airplane in question. Among the factors affecting the values of these derivatives are wing and fuselage forms and interference between the various parts of the airplane.

The effect of various tip shapes and amounts of dihedral for a rectangular Clark Y wing are given in refer-
A theoretical prediction of some of the lateral-stability characteristics for wings is given in reference 3. The present investigation of N.A.C.A. 23012 wings was made to determine the effect of taper, sweep, dihedral, and partial-span split flaps on the lateral-stability characteristics that depend upon sideslip.

Lateral-stability characteristics of the wings and equations for computing approximations to these characteristics are given in the report. The force and the moment coefficients are also included.

APPARATUS AND MODELS

The tests were made in the N.A.C.A. 7- by 10-foot wind tunnel with the regular 6-component balance. The closed-throat tunnel is described in reference 4 and the balance is described in reference 5.

The models are made of laminated mahogany to the N.A.C.A. 23012 profile. Plan-form and elevation drawings of the wings are given in figure 1 and a photograph of one of the tapered wings mounted on the balance is given in figure 3.

The tip plan form of the rectangular wing is composed of two quadrants of similar ellipses; for the tapered wings, the ordinates of the ellipses have been expanded in proportion to the taper of the individual leading or trailing edges. The N.A.C.A. 23012 profile is maintained to the ends of the wings and, in elevation, the maximum upper-surface section ordinates are in one plane. The area of the rectangular wing is 3.917 square feet and the aspect ratio is 6.383; for the tapered wings, the area is 4.101 square feet and the aspect ratio is 6.097.

The split flaps are made of 1/16-inch steel plates attached to the wing at an angle of 60°. The 20-percent-chord split flaps are tapered with the wing chord and extend over 60 percent of the span at the center section, which is believed to be representative of modern practice.

TESTS

Each wing was tested with 0° and 5° dihedral and with
the partial-span split flaps deflected $0^\circ$ and $60^\circ$. Each wing combination was tested at angles of yaw of $-50^\circ$, $0^\circ$, $20^\circ$, $50^\circ$, $100^\circ$, and $150^\circ$. At each angle of yaw, tests were made at angles of attack of $-10^\circ$, $-5^\circ$, $0^\circ$, $5^\circ$, $10^\circ$, $15^\circ$, $18^\circ$, and $20^\circ$. At zero yaw, enough tests were made at additional angles of attack to obtain the values of minimum drag and maximum lift. For each test, all six components of force and moment were measured.

All tests were made at a dynamic pressure of 16.37 pounds per square foot, which corresponds to an air speed of about 80 miles per hour under standard conditions. The Reynolds Number of the tests was about 609,000 based on the average chord of the wings. The effective Reynolds Number, based on a turbulence factor of 1.6 for the tunnel, was about 974,000.

RESULTS

The data, which refer to wind axes, are given in standard nondimensional coefficient form as follows:

- $C_L$, lift coefficient, $L/qS$.
- $C_Y$, lateral force coefficient, $Y'/qS$.
- $C_D$, drag coefficient, $D/qS$.
- $C_L'$, rolling-moment coefficient, $L'/qS_b$.
- $C_m$, pitching-moment coefficient, $M/qSc$.
- $C_n'$, yawing-moment coefficient, $N'/qS_b$.

where $L$ is the lift.

- $Y'$, lateral force.
- $D$, drag.
- $L'$, rolling moment.
- $M$, pitching moment.
- $N'$, yawing moment.


\( q \), dynamic pressure, \( 1/2 \rho V^2 \).

\( V \), tunnel-air velocity.

\( \rho \), air density.

\( S \), wing area.

\( c \), average wing chord.

\( b \), wing span.

and

\( \alpha \) is the angle of attack.

\( \psi \), angle of yaw, degrees (positive when the wing is yawed to the right).

\( \Lambda \), sweep angle, degrees (positive when the line of section quarter-chord points is swept back).

\( \Gamma \), dihedral angle of the plane of the section chord lines exclusive of the tip portion.

\( \Gamma_0 \), effective dihedral angle of the wing including the tip portion. (Reference 2 indicates that the rounded tip with the maximum upper-surface ordinate points in one plane gives an effective dihedral angle of \( 1^\circ \).)

All the forces and moments have been given with respect to the wind-tunnel system of axes that intersect in the wing at the center of the balance support, which is very nearly the aerodynamic center of the wings without flap or dihedral. The center of the balance support was located on the chord line of the center sections and back of the leading edge 23.10 percent of the chord for the rectangular wing; 16.61 percent of the root chord for the tapered wing with \( \Lambda = -4.75^\circ \); 30.59 percent for \( \Lambda = 4.75^\circ \); and 44.50 percent for \( \Lambda = 14.00^\circ \).

The lift, the drag, and the pitching moments at \( 0^\circ \) yaw were corrected for tunnel effects to aspect ratio 6 in free air. These data are given in figures 3 to 6 to show the aerodynamic characteristics of the models.

The cross-wind force, the rolling moment, and the
yawing moment were corrected for initial asymmetry by deducting the values obtained without yaw from the values obtained with yaw. The coefficients were plotted against angle of yaw; figures 7 to 9 are sample plots.

The stability characteristics, \( \left( \frac{dC_l'}{d\psi} \right)_o \), \( \left( \frac{dC_m'}{d\psi} \right)_o \)

and \( \left( \frac{dC_y'}{d\psi} \right)_o \), were obtained by measuring at zero yaw the slope of the curves of the coefficients against angle of yaw. Figures 10 to 12 give the variation of the stability characteristics with angle of attack. For the angles of attack at and above the stall, the values become indeterminate, as is indicated by some of the curves in figures 7 to 10.

The variations in \( \left( \frac{dC_l'}{d\psi} \right)_o \) with \( \alpha \) are given in figure 10(a) to (d) so that a direct comparison may be made with the data of previous investigations.

The angles of attack, yaw, and dihedral are believed to be within ±0.1° of the values given. Experimental errors in the coefficients are believed to be within the following limits:

\[ C_L, \pm 0.005 \text{ (for maximum lift)} \]
\[ C_D, \pm 0.0005 \text{ (for minimum drag)} \]
\[ C_Y', \pm 0.001 \]
\[ C_{l}', \pm 0.001 \]
\[ C_{m}, \pm 0.002 \]
\[ C_{n}', \pm 0.001 \]

**DISCUSSION**

It was found that the values of \( \left( \frac{dC_l'}{d\psi} \right)_o \) could be
represented by an empirical equation. The equation is

\[
\frac{dC_L}{d\psi} = 0.00021 \Gamma_0 - 0.0018 \frac{c_s}{b} (1 - \lambda) + 0.000056 \left[ \Lambda + 5.70 - 2.1 \frac{c_s}{b} (1 - \lambda) \right] \left[ C_L + 0.2 \right]
\]

(Note that \( \Gamma_0 \) is the effective dihedral angle of the wing, as previously explained.)

where \( c_s \) is the root chord of the wing,

\( \lambda \), taper ratio (\( \lambda = 1/3 \) for the 3:1 tapered wings),

and \( b \), the wing span.

In figure 10(e) and (f) is shown the agreement of the test points and the curves calculated from the equation.

The equation indicates that the effect of a change in dihedral on \( \frac{dC_L}{d\psi} \) is independent of taper, sweep angle, and split-flap deflection. The coefficient of the \( \Gamma \) term is practically the same as that obtained for other sections and plan forms (reference 2), and is slightly less than that obtained by theory (reference 3). Inasmuch as the value of \( \frac{dC_L}{d\psi} \) for a given value of \( C_L \) is practically unaffected by split flaps, airfoil section might be expected to have little effect on \( \frac{dC_L}{d\psi} \). The variation of \( \frac{dC_L}{d\psi} \) with \( C_L \) and \( \Lambda \) was not predicted by the theory of reference 3.

The values of \( \frac{dC_m}{d\psi} \) are comparatively small and
show no consistent variation with $\Gamma$, $\Lambda$, or taper. (See fig. 11.) There is a tendency for \( \frac{dC_n}{d\psi} \) to have the smallest numerical values at small angles of attack and for the values to be algebraically less for the wings with flaps deflected than with flaps neutral. Although the curves in figure 11 are similar to those for the Clark Y wing given in reference 2, there is no indication of the effect of dihedral on \( \frac{dC_n}{d\psi} \), which was given therein.

The values of \( \frac{dC_Y}{d\psi} \) given in figure 12 may be represented fairly accurately, for angles of attack below the stall, by the following empirical equations:

For the wing without flaps,

\[
\left( \frac{dC_Y}{d\psi} \right)_0 = 0.012 C_D - 0.00011 \Gamma_0
\]

and, for the wing with flaps deflected 60°,

\[
\left( \frac{dC_Y}{d\psi} \right)_0 = 0.012 C_D - \{0.0000065 \left[ 82 \frac{c_s}{b} (1 - \lambda) - \Lambda \right] + 0.00011 \} \Gamma_0
\]

The effect of $C_D$ on \( \frac{dC_Y}{d\psi} \) is practically independent of taper, flaps, and dihedral.

CONCLUDING REMARKS

The results of this investigation indicate that:

1. The effect of increasing the dihedral is to increase \( \frac{dC_Y}{d\psi} \) by an amount equal to 0.00021 per de-
The effect of deflecting 60-percent-span, 20-percent-chord, split flaps 60° is, generally, algebraically to decrease \( \frac{dC_n'}{d\psi'} \), to increase the effect of sweepback on \( \frac{dC_L'}{d\psi'} \), and to produce no change in \( \frac{dC_L'}{d\psi'} \) for a given value of \( C_L \).

4. Increasing the sweepback increases the effect of \( C_L \) on \( \frac{dC_L'}{d\psi'} \) and of dihedral on \( \frac{dC_y'}{d\psi'} \).

5. A change in airfoil section is believed to have little effect on \( \frac{dC_L'}{d\psi'} \) for a given value of \( C_L \).

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National Advisory Committee for Aeronautics,
Langley Field, Va., March 2, 1939.
REFERENCES


Note:
A and B are quadrants of similar ellipses.

Figure 1. Plan and elevation drawings of the N.A.C.A. 23012 wings.
Figure 2.—Photograph of a tapered wing on the balance in the 7-by 10-foot wind tunnel. Split flap deflected.
Figure 3. Aerodynamic characteristics of rectangular N.A.O.A. AS012 rounded-tip wing.

Figure 4. Aerodynamic characteristics of 311 tapered N.A.O.A. rounded-tip wing, $\alpha = 14.00^\circ$.

Figure 5. Aerodynamic characteristics of 311 tapered N.A.O.A. 23012 rounded-tip wing, $\alpha = 4.75^\circ$.

Figure 6. Aerodynamic characteristics of 311 tapered N.A.O.A. 23012 rounded-tip wing, $\alpha = -4.75^\circ$. 
Figure 7.- Variation of rolling-moment coefficient with yaw. Rectangular N.A.O.A. 25012 rounded-tip wing.
Figure 8. Variation of yawing-moment coefficient with angle of yaw. Rectangular N.A.C.A. 23012 rounded-tip wing.
Figure 9 - Variation of lateral-force coefficient with yaw. Rectangular N.A.C.A. 23018 rounded-tip wing.
Figure 10a, b, c, d - Rate of change of rolling-moment coefficient with angle of attack at zero yaw. N.A.C.A. 23012 rounded-tip wings.
Figure 10a-f, 11, 12

Figure 10a-f - Rate of change of rolling moment coefficient with angle of attack at zero yaw. N.A.C.A. 23012 rounded-tip wings.

For key see Fig. 11

Figure 11 - Rate of change of yawing moment coefficient with angle of attack at zero yaw. N.A.C.A. 23012 rounded-tip wings.

Figure 12 - Rate of change of lateral force coefficient with angle of attack at zero yaw. N.A.C.A. 23012 rounded-tip wings.