AIAA/CEAS 98-2260

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4th AIAA/CEAS Aeroacoustics Conference
June 2-4, 1998 / Toulouse, France

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SCREWS, PROPELLERS AND FANS BASED ON THE MOBIUS STRIP

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ABSTRACT

A Möbius strip concept is intended for improving the working efficiency of propellers and screws. Applications involve cooling, boat propellers, mixing in appliances, blenders, and helicopters. Several Möbius shaped screws for the average size kitchen mixers have been made and tested. The tests have shown that the mixer with the Möbius shaped screw pair is most efficient, and saves more than 30% of the electric power by comparison with the standard. The created video film about these tests illustrates efficiency of Möbius shaped screws.

1. INTRODUCTION

Several new methods for improving the working efficiency of propellers and screws and the mixing of exhaust jets with ambient air have proposed by the two first authors of this paper in 1993. The new concept for improving the working efficiency of propellers and screws was formulated in final version in the patent application to NASA [1]. NASA Technical Briefings has published the note [2] in 1996, which presents the short description of this invention. All these proposals are based on the untraditional designs which provide more efficient mixing of two flows. In general, this also favors noise reduction and thrust augmentation, and can be applied for the rocket, aviation, domestic and other industries. The theoretical and experimental research results for several nozzle designs (Bluebell, Chisel, Telescope shaped nozzles, corrugated Co-Annular nozzles, Co-Annular nozzle with Screwdriver Centerbody) are promising (see, for example, the papers [3-5]). There is only the preliminary theoretical and experimental results of efficiency of the Möbius shaped propellers and screws. In particular, the significant effect is observed in the small scale devices with screws like Möbius strip: coffee-beans, mixers (the unpublished experimental results conducted in Russia (1991) by Prof. Veniamin Maron, Mikhail Gilinsky with colleagues). The same general idea unites "A Bluebell nozzle" and "A Möbius strip" concepts: to prefer curvilinear surfaces with significant change of a curvature along a flow by comparison with the traditional axisymmetric or plane surfaces. Therefore some research results for the first designs can be used for improving of the second and vice versa.

The new concept for improving the working efficiency of propellers and screws has proposed in the invention disclosure in 1993. NASA Technical Briefings has published the note [2], 1996, which presents the short description of this invention. The last version of this invention is in NASA patent application [1]. The concept is based on the Möbius strip-one-side surfaces. There are several embodiments of such shapes. Each of them can be optimal for different application in different surrounding media.

Mainly, in this paper the perspective research plan is presented which should be fulfilled as a part of a long term projects under several NASA and CRDF grants for Hampton University. These grants are directed for creation of a Fluid Mechanics and Acoustics Laboratory at Hampton University for research, training and teaching minority students to enhance the motivation of students within scientific areas and problems important for NASA, aviation and domestic industries. These projects are conducting within a Partnership between Hampton University and NASA Langley Research Center.

The research includes an analysis of the Möbius elliptical screw modification in the air flows. Definition of the optimal parameters provided maximum of thrust, lift, mixing, or minimal noise should be obtained by the analytical theory, numerical simulation and experimental tests. These research results will be also applied for sport goals by improving of the sail or oar shapes. The final conclusions are drawn by numerical simulation of 2D or 3D flow around the rotated screws or blades and experimental tests in the NASA wind tunnels and in the natural conditions using small scale models (toys, sport vehicles etc.). In the theory we will use a perfect gas model, Euler or Navier-Stokes equations with the several turbulent models and several packages of CFD codes, CFL3D [6], ADPAC [7], or ROTOR [8], involving the specialists-users of these codes in joint research for solution of these complicated 3D problems. Also we will use approximate methods for rotating blades using the big experience in this field described in the reports [9,10].

The similar research for some screw modifications in liquids will be also analyzed. These liquids are water,
glycerine, viscous-plastic medium, which can be applied to enhance efficiency of the screws for boats and mixers. And, of course, it is very important and interesting an analysis of application such shapes in dispersive and dry medium like sand, cement powders, coffee-beans and different mixtures.

It is possible to apply other corrugated shapes with more complicated analytical representation than elliptical or super-elliptical Möbius shapes. However, elliptical shapes are very convenient for theoretical description of this phenomenon.

II. MOBIUS-SHAPED EMBODIMENTS

2.1 Möbius Strip Definition and Motivation

We have generalized Möbius strip concept and proposed the several new modifications for different applications. The main goal of the proposed design is to reduce a rotated element drag and, simultaneously, to increase the area for capture of the still medium without increasing the power needed for rotation. Such surface can be the Möbius strip which is one-sided surface. The Möbius strip has been proposed as the basis for optimally shaped airplane and boat propellers, fans, helicopter rotors, mixing screws, coffee grinders, and concrete mixers. The ambient medium can be air, water, concrete, coffee beans etc. Conventional (non-Möbius) devices of this type consist mostly of two-sided blades, which are not always optimal.

In Figures 1 and 2 the two limited cases are considered for a rotating round ring with an infinitesimal thickness around Z-axis. In the first case a ring has a cylindrical surface with X-axis as an axis of symmetry (a lateral rotation). In the second case a ring is located in meridional plane which revolves around Z-axis together with a ring (a frontal rotation). In the first case a medium drag is minimal as well as is minimal medium capture that provides mixing efficiency, and, vice versa, in the second case a medium drag and mixing efficiency are maximal. So, a drag is ziro at the lateral rotation with neglecting of viscous effects. At the frontal rotation a drag can be calculated using rough approximate analytical approach or numerical methods. For example, a drag ring can be calculated the simple approximation.

Let us assume that at the frontal steady rotation all kinetic flow energy (in an inverse motion) completely transfer to a potential energy at the ring surface, i.e. to a total flow pressure. In other words, we will neglect of velocity squared by comparison with pressure. A pressure behind frontal ring side is assumed equal constant pressure in surrounding medium, $p_\infty$. This corresponds assumption that this is a cavity region with constant pressure as in the case of blunt bodies moving in an incompressible liquid with a big enough speed. Than, introduce a drag coefficient, $c_d$, as a ratio between a full force, $F$, to the ring area, $S_r = \pi(R_2^2 - R_1^2)$ and a maximal dynamic force to the frontal ring area, $1/2\rho_\infty V_\infty^2 S_r$, which is produced by flow with the maximal linear ring velocity, $V_\infty = \Omega R_2$. Here, $R_1$ and $R_2$, are internal and external ring radii, $\rho_\infty$ is medium density, $\Omega$ is rotational ring speed. We have the following relationships:

$$c_p(R, \phi) = k_0(p(R, \phi) - p_\infty), k_0 = \frac{1}{1/2\rho_\infty \Omega^2 R_2^2}$$

$$c_d = \frac{1}{S_r} \int_0^{2\pi} \int_{R_1}^{R_2} c_p R dR d\phi = \frac{1}{4}(1 + R_1^2/R_2^2)$$

where we have used that $p - p_\infty = \rho_\infty \Omega^2 R^2 \sin^2 \phi$. On the face on it the last formula (2) for drag coefficient, $c_d$, is paradoxical: with increase of internal radius, $R_1$, i.e. with decrease of ring wide, a drag coefficient increases. This effect is connected with decrease of ring area and increase of average flow velocity. Recall that a local flow velocity increases linearly along a radius. in the limited case, $R_1 = R_2$, a drag coefficient, $c_d = 1/2$, that equal the drag coefficient for a round frontal located disk in hypersonic flow in accordance with hypersonic Newton theory. It is not small wonder because we used the same assumption that in this theory. Justification of such approach can be analytical Gonor's theory [15] which has shown that Newtonian impact formula for drag coefficient,

$$c_p = 1 - k^2 \cos \theta$$

is exact even for incompressible inviscid flows around some blunt bodies such as an elliptical cylinder, ellipsoid of revolution, and three-axial ellipsoid. Here, $\theta$ is a local angle of flow velocity vector with a tangent plane to the surface in considered surface point. Comparison of exact numerical solutions with the solution using the formula (3) are shown a very good agreement for wide class of other bodies.

The analogical formula with (2) can be given in the quadratures for elliptic ring but this formula is more cumbersome, and we omit it in this paper, as well as the formula for a drag coefficient of a Möbius screw which shown in Figure 4. Calculation of this coefficient is conducted using simple single integration numerically. This is a subject of other paper. Here note only, that drag of a round Möbius-shaped element $A_0 A_N B_0 B_N$, in Figure 4 (i.e. one petal of the Möbius screw located in the first quarter of the Cartesian coordinate system OXYZ) has a drag less than frontal rotating ring over ~40% for the same rotation speed. This can be also explained by less common frontal area for Möbius element by comparison with frontal ring. This comparison was made for the same area of both designs.

Thus there is some intermediate ring shape which provide compromising values for medium drag and capture. One such possible shape is a Möbius Strip shown
in Figure 3. This strip at the Z-axis in the its top has a shape close to frontal rotating ring, and lower transit to the lateral rotating ring. There are many different modifications of Möbius similar surfaces with different analytical representations. Below some of its will be illustrated.

A Möbius strip is made by giving a half twist to a strip of elastic material, then joining the ends to obtain a smooth surface. This design is one-side in the sense that in principle, one can trace out a continuous line along the strip from any point on its surface to any other point on the surface, without leaving (for example, through a border) or penetrating the surface. The one-sided, smooth shape of a Möbius strip provides a large capture area while generating the least possible turbulence in three-dimensional flow, and thus maximized working efficiency.

Propeller shapes based on the Möbius strip, and their orientations with respect to axes of rotation, can be varied and extended to suit the requirements of specific applications. For example, a propeller or fan blade could be made as a single basic Möbius strip shape as in Figures 5 and 6, where the strip can be rotated around Z axis. Figure 6 shows another example, in which the ends of a single strip were twisted and joined at an axis of rotation to form two fan blades, each of which is the equivalent of a single Möbius strip that has been folded and joined to the axis at the fold. Other potential variations include multiple strips fastened in the same plane or different planes, rotating about the same axis or different axes; and strips made wholly or partly of circular (Figure 5), elliptical (Figure 6), super-elliptical (Figure 10) or otherwise curved sections (Figure 9). Corrugated front or back edges of the Möbius strip (or both edges) can be applied to enhance mixing (Figure 11). Shown above screws in Figures 6 and 12 are preferable for propellers and mixers. If we will use only upper part of these embodiments then the screws are preferable for producing thrust and applicable for boat-screw, fans and others. The two elliptic Möbius shaped modification are shown in Figures 7 and 8. Of course, these examples are only illustrative pictures, which clarify our approach. In reality, instead of an infinitesimal thin screw surface should be applied a wing shaped screw. The optimal geometrical parameters of the screw as well as its cross section can be different for different applications.

2.2 The Main Design Description

The simpliest Möbius shaped surface can be described by super-elliptical equation. Recall the super-elliptical 2D contour is described, for example, in the plane XY by the equation:

\[(X/a)^n + (Y/b)^n = 1\]  (4)

where a, b are extremal values of radius-vector modulus of this curve (known as half-axes). For n=2 the super-ellipse is the usual ellipse, and when exponent n increase to infinity the super-ellipse transforms to rectangular.

A Mobius super-elliptical screw modification can be constructed following method. Our description will explain with the references to Figure 4. Let we draft ellipse (or circle) with origin O of a Cartesian coordinates XYZ in the plane Z=0. Call this ellipse as “based ellipse” with the half-axes \(a_0, b_0\). Let the interval \(A_0A_N\) of Z-axis is divided by the N equal sub-intervals, and some \(a_0\)-arc of the based ellipse \(B_0B_N\) also is divided by the same number N of the equal sub-arcs. The surface of the screw can be made from super-ellipses which join in consecutive order end-points \(A_i\) \((i=0,1,...N)\) of the vertical sub-intervals on the Z-axis with the end-points \(B_i\) \((i=0,1,...N)\) of the based ellipse sub-arcs on the horizontal plane Z=0. The screw surface can be described in Cartesian coordinates:

\[z = b_i(1 - x^n/a_0^n - y^n/b_0^n)^{1/n}\]  (5)

\[b_i = b_0 + b_N - b_0 \cdot \arctan \frac{y}{x}{\varphi_{\text{max}}}\]  (6)

where \(n=n(\varphi)\) or in cylindrical coordinates \(r, \varphi, z:\)

\[z = [b_0 + (b_N - b_0) \cdot \frac{\varphi}{\varphi_{\text{max}}}] [1 - r^n (\cos^n \varphi + \sin^n \varphi)^{1/n}]\]  (7)

where \(b_0 = |OB_0|, b_N = |OB_N|,\) and \(\varphi_{\text{max}}\) is the maximal azimuthal angle of the petal, which can be changed along the petal edge, for example, sinusoidaly as for a Bluebell nozzle design. In simpliest case this change can be described by the formula:

\[\varphi_{\text{max}} = (\varphi_{\text{max}})_0 [1 + \delta \cos(n_c \frac{s}{s_{\text{sm}}} \pi/2)]\]  (8)

where \(s, s_{\text{sm}}\) are the current and maximal length of the super-elliptic initial petal edge (i.e. the curve \(A_NB_N\) in Figure 4), or the corresponding values of the areas sectors under this curve, or square root of this areas. This allows to form corrugated edge uniformly then using usual dependence of the polar angle in the plane XOZ. Note, the exponent \(n\) also can depend of the azimuthal angle, which provides to change super-ellipticity downstream of the screw surface.

Thus in the case of an invariable edge and super-ellipticity a surface shape depends from six parameters: two half-axes, \(a_0, b_0,\) and exponent n, two end-point coordinates \(b_0, b_N\) and the length of the arc \(l_k = |\sim B_0B_N|\). Let \(a_0=1,\) i.e. \(a_0\) is a characteristic length. Then we can characterize screw geometry by four main parameters: a ratio of the general axes of the super-elliptic -c=\(a_1/b_1,\) exponent n, the vertical Z-interval length,
h=\vert A_x A_y \vert$, and $\alpha^\ast$-arc (or its length). Below we suppose that a based ellipse is a circle, so that $A_x = B_y = 1$.

In this surface the vertical oriented region at the $Z$-axis smoothly transits to horizontal oriented region, and a flow at this solid surface also follows of such change.

We can use several designs as described above which are mounted to the same vertical interval (in practice, to the same part of the axial cylindrical shaft). Symmetrical located several designs around axis of rotation $Z$ produce the thrust or lift as in the case of a usual wing or propeller. Several M\(\ddot{o}\)bius screw modifications with the three and four symmetrical elements (petals) illustrate Figures 7 and 8. If, for example, to mount two 2-petal screws antisymmetrical relatively of XOY plane, then a screws as are shown in Figures 14a,\(\ddot{f}\) doesn't produce the thrust, but enhances mixing. In this case the design can be applied in different mixers. Possible also a set of the elements located along the axis of rotation $Z$ and mounted at the consecutive intervals as well as symmetrically and anti-symmetrically.

In Figure 9a almost arbitrary complicated M\(\ddot{o}\)bius-shaped screw for mixers is shown. Such a surface can be constructed and analytical described using different combinations of smooth 3D surfaces based on the set of space lines. We construct these lines using ellipse and parabola as a projection of this space curve to the coordinate plane XOY. So that giving the coordinates of the fixed extremal points of the needed surface, $A(0,0,z_A)$, $M(z_M,y_M,z_M)$, and $B(z_B,0,0)$ (see Figure 9b), we can get this space curve equation in parametric shape as:

$$y = y_M[1-(z-z_M)/a_y]^{1/n}$$

$$z = z_M[1+\sqrt{1-y/y_M}]$$

if $0 < z < z_M$, $z_A < z < z_M$

$$y = y_M[1-(z/z_M-1)^2]$$

$$z = z_M+b_x[1-y^m/y_M^{m}]^{1/m}$$

if $z_M < z < z_B$, $0 < z < z_M$

where $a_x = z_A - z_M$, $b_x = z_B - z_M$, and $n,m=2,4,...$

For continuation of this curve in the antisymmetry quarter of a Cartesian coordinate system ($z > 0$, $y < 0$, $z < 0$) we use antisymmetric reflection relative bend point $B$. Based on this analytical representation several different screws for kitchen mixers were made and tested at the NASA LaRC.

We expect to obtain essential working efficiency of propellers and screws, and for different applications to obtain noise reduction and thrust augmentation in a wide region of Mach and Reynolds and numbers for aviation, domestic and other industries.

2.3 M\(\ddot{o}\)bius-shaped screws for mixers.

All industrial companies, who are interested in the invention [1], agree to participate in funding of research, development and joint marketing. However, these companies require the preliminary experimental and numerical simulation proofs, that it can be effective and adopted to the company's product. Large scale experiments with the proposed design are very expensive for us in this time. As yet, we have only been able to conduct cheap and simple tests. Several M\(\ddot{o}\)bius shaped screws for the middle class of the kitchen mixers have made and tested. Some of them are shown in Fig.14. These pairs are mounted in the mixer, which was established on the top of the vessels with water and small plastic particles on the bottom. The screws and corresponding equipment are shown in Fig.14a-f for the small vessel (b,c) and bigger vessel (d,e). The rotation speed of the screws increased smoothly by the voltage regulator, and power expenditure was measured by the Digital Wattmeter. These are shown on the right and left of the vessel respectively in Figure 14b-d. When rotation speed mount to the definite value the particles acquire a motion, go up and involve in vortex motion with water, and mixing process arises. The power value corresponding this mixing start characterizes efficiency of the mixer. The tests have shown that the mixer with the M\(\ddot{o}\)bius shaped screw pair (right in Figure 14a or left in Figure 14f) is most efficient, and saves more than 30\%, of the electric power by comparison with the standard (left in Figure 14a). The video film about these tests was created and it clearly demonstrate M\(\ddot{o}\)bius-shaped screws efficiency. It can be used with the scientific-popular goals, for teaching, and with commercial goals. In particular, the discovered effect can be applied in the manufacture of liquid semiconductors.

III. CONCLUSIONS

New M\(\ddot{o}\)bius strip concept is intended for improving the working efficiency of propellers and screws. Applications involve cooling, boat propellers, mixing in appliance, blenders, and helicopters. Several M\(\ddot{o}\)bius shaped screws for the average size kitchen mixers have been made and tested. The tests have shown that the mixer with the M\(\ddot{o}\)bius shaped screw pair is most efficient, and saves more than 30\% of the electric power by comparison with the standard. The created video film about these tests illustrates efficiency of M\(\ddot{o}\)bius shaped screws.

IV. ACKNOWLEDGEMENTS

We would like to acknowledge the Jet Noise Team support and help, to thank Dr. Dennis Bushnell for his attention and interest to our research and useful suggestions.
V. REFERENCES


Fig. 1 Limited case 1: Lateral rotation of the round ring at the Z-axis. Drag and mixing efficiency are minimal.

Fig. 2 Limited case 2: Frontal rotation of the round ring at the Z-axis. Drag and mixing efficiency are maximal.

Fig. 3 The Round Möbius-shaped ring rotating at the Z-axis. Drag and mixing efficiency are intermediate.

Fig. 4 The two-petal elliptic screw with rotation at the Z-axis. The surface is constructed by ellipses which join the points, $A_i$, of the Z-axis interval, $[A_o, A_N]$, with the points, $B_i$, of the arc, $(B_o, B_N)$, of the ellipse in XOY-plane.
MOBIUS SCREW MODIFICATIONS

**Round Mobius-Shaped Screw**

Figure 5

**Elliptic Mobius-Shaped Propeller**

Figure 6

\[ a_1 = 4, \ b_1 = 1 \]

**3-Petal Mobius-Shaped Screw**

Figure 7

**4-Petal Mobius-Shaped Screw**

Figure 8

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**Fig. 5** The round Möbius-shaped screw with a cylindric holder for mixers.

**Fig. 6** The elliptic Möbius-shaped propeller with the half-axes \( a_1 = 4, \) and \( b_1 = 1. \)

**Fig. 7** The 3-petal Möbius-shaped round screw for motor-boats or propeller for airplanes creating thrust.

**Fig. 8** The 4-petal Möbius-shaped round screw for motor-boats or propeller for airplanes creating thrust.
Fig. 9 The 2-petal Möbius-shaped screw with a cylindric holder for mixers. The contours are constructed using an ellipse parabola combination for space curve projection to YOZ and XOY planes. a) A common view. b) The schematical draft; A, M, and B are extremal points of the space curve described by the equations (9-10).

Fig. 10 The super-elliptic Möbius-shaped screw with a cylindric holder for mixers. The contours are constructed using equations (7,8) with n=5, a=b=1, \( b_x=2.5 \), and \( b_y=3.0 \). a) 2-petal screw (m=2). b) 4-petal screw (m=4).
**Fig.11** The corrugated 2-petal round Möbius-shaped screw with a cylindric holder for mixers. The lip-line edge has a sinusoidal shape and rotation is around the Z-axis.

**Fig.12** The 2-petal Möbius-shaped blade for fans. The blade is made from a thin steel strip and this design was tested at the NASA LaRC. Only preliminary results are currently available.

**Fig.13** The super-elliptic sail with the based circle. The sail can be turned along the based circle around the cylindrical holder for an optimal thrust depending on wind direction.
Fig. 14 - Möbius-Shaped Screws for Mixers.
a) Picture of the three pair tested screws. Standard pair is on the left; b) Equipment for tests: voltage regulator (left), mixer over the small vessel with water, plastic particles, and the 1st Möbius screw pair (center), digital wattmeter (right). c) The same as in b) but with the 2nd Möbius screw pair. d) The same as in b) but with bigger vessel. e) Big vessel with the 1st Möbius screw pair during the test. f) Screws and a standard mixer used in tests.